

# Analytical friction model for sliding bodies with coupled longitudinal and transverse vibration

P. Udaykant Jadav<sup>1,3,\*</sup>, R. Amali<sup>1</sup>, O.B. Adetoro<sup>2</sup>

<sup>1</sup>) Department of Engineering Design and Mathematics, University of the West of England, Bristol, BS16 1QY, United Kingdom.

<sup>2</sup>) Department of Mechanical and Aerospace Engineering, Brunel University London, Uxbridge, UB8 3PH, United Kingdom.

<sup>3</sup>) Rotork Controls Ltd., Bath, BA1 3JQ, United Kingdom.

\*Corresponding e-mail: Priyang.Jadav@rotork.com

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**ABSTRACT** – An analytical friction model is developed to compute the drive force required to slide a body over a surface subjected to coupled longitudinal and transverse vibration. Previously, this computation was only possible under either longitudinal or transverse vibration using a separate analytical model for each mode. The model proposed in this paper is applied to longitudinal, transverse, and coupled vibration modes. Results are in good agreement with existing literature. Analytical results are replicated numerically by use of a specially developed friction subroutine which can be integrated into any Abaqus® dry contact simulation.

## 1. INTRODUCTION

The efficiency of mechanical systems that involve predominantly dry sliding contacts can be significantly improved by friction reduction via the use of conventional lubricants or surface coatings. A less typical method, and one that has been the subject of theoretical analyses for several decades, is the phenomenon of reduced friction force of surfaces when subjected to vibration. In such studies vibration is applied by exciting one of the contacting bodies either in the longitudinal [1], transverse [2] or normal [3] direction.

In recent research Gutowski and Leus [1-2] have shown that good agreement between analytical and experimental results can be achieved by utilising models that have dynamic equations of motion, while also including terms to describe compliance of both the contact and the drive system. A model for longitudinal vibration and a separate model for transverse vibration were developed [1-2], however, a model for coupled longitudinal and transverse vibration does not exist in literature. This paper proposes a new friction model to describe changes in friction force and drive force for sliding surfaces subjected to coupled longitudinal and transverse vibration. The new model is validated against previous models [1-2] that have shown good agreement with experiments.

## 2. METHODOLOGY

The domain, Figure 1, consists of a body of known mass  $m$  moved over a base that can be oscillated sinusoidally such that the longitudinal and transverse components of motion,  $x_b$  and  $y_b$ , act simultaneously and in phase. Mathematically this coupled motion can

be treated as vibration applied along an axis which is at angle  $\theta$  relative to direction  $X$ , Figure 2. Movement of mass  $m$  over the base is imposed by a constant drive velocity  $v_d$  applied at point  $B$ , while vibration of the base is also transferred to the body. This corresponds to an instantaneous external drive force  $F_d$  that is transferred to point  $A$  of the body via a mechanical drive system of which the stiffness  $k_d$  is known. No structural damping of the drive system is assumed, hence  $h_d = 0$ .

Contact compliance is included by assuming Dahl friction behaviour [4]. The deformable contacting asperities of the sliding body and base are modelled as a single lumped asperity  $MN$ . Endpoint  $N$  is attached to the sliding body, while the free endpoint  $M$  interfaces with the vibrating base. During vibration, change in relative displacement of  $M$  and  $N$  creates change in magnitude  $s$  and direction  $\beta$  of the contact's elastic deformation. This causes changes in the friction force between the two bodies.

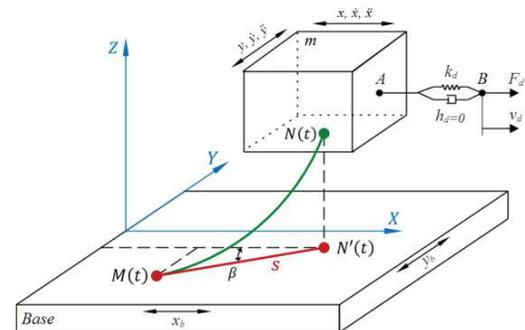


Figure 1 Schematic of domain [2] showing coupled vibration with base motion components  $x_b$  and  $y_b$

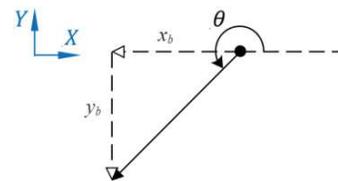


Figure 2 Mode of vibration defined by angle  $\theta$ .

The new analytical model is a development of previous models [1-2] and is described by a mathematical computational procedure developed in Matlab®/Simulink®. The model enables computation of

$F_d$  under different conditions, with the variable inputs being mass  $m$ , drive velocity  $v_d$ , amplitude of vibration velocity  $v_a$ , vibration frequency  $f$ , contact normal force  $F_N$ , coefficient of friction  $\mu$ , contact stiffness  $k_t$ , drive system stiffness  $k_d$ , and angle  $\theta$ .

The same domain has also been generated by finite element method in Abaqus® and the new model has been implemented via a specially developed friction subroutine. The base orientation can be set between  $0^\circ \leq \theta < 360^\circ$  in the  $XY$  plane to apply vibration in any mode.

**3. RESULTS AND DISCUSSION**

Results of the new model when set to longitudinal vibration mode,  $\theta = 0^\circ$ , are plotted in Figure 3, and with transverse vibration mode,  $\theta = 90^\circ$ , in Figure 4. Results of previous models are overlaid for validation. Simulations begin with the body stationary, hence  $F_d = 0\text{N}$  at  $t = 0\text{s}$ . Constant drive velocity  $v_d$  at  $t > 0\text{s}$  causes elastic deformation  $s$  to increase, resulting in steady rise of  $F_d$  until  $t \approx 0.04\text{s}$ . At this time the magnitude of  $F_d$  is large enough to cause the body to breakaway. At  $t = 0.05\text{s}$ , switching on vibration significantly reduces the magnitude of  $F_d$  as the body continues to slide.

A characteristic of the new model when evaluated in Matlab®/Simulink® is that it produces a continuous small undulation of  $F_d$  in longitudinal mode (Figure 3). The undulation diminishes as vibration mode approaches transverse, Figure 4.

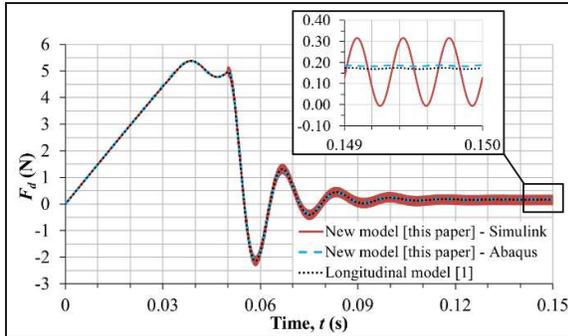


Figure 3 Validation of longitudinal vibration results.

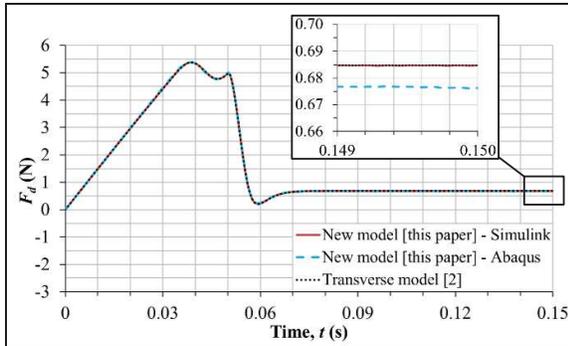


Figure 4 Validation of transverse vibration results.

Further simulations in Abaqus®, Figure 5, illustrate the influence of  $\theta$  and  $k_v$  on  $F_{dv} / F_{ds}$ , where  $k_v = v_a/v_d$ , and  $F_{dv} / F_{ds}$  is the drive force under vibration in relation to drive force without vibration.

Each data point in Figure 5 corresponds to the result of a single simulation. For each value of  $k_v$ , seven simulation results in the range  $0^\circ \leq \theta \leq 90^\circ$  allow a trend to be established. Trends for different values of  $k_v$  are indicated by dashed lines. A reduced number of simulations at  $\theta > 90^\circ$  agree with the expected trends. Increasing  $k_v$  decreases the drive force in all vibration modes. The greatest reduction in drive force is always achieved by longitudinal vibration ( $\theta = 0^\circ, 180^\circ$ ) and the curve is always symmetrical about  $\theta = 180^\circ$ . The same result can thus be obtained at multiple values of  $\theta$ .

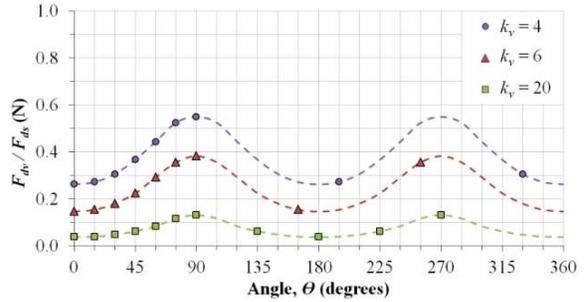


Figure 5 Influence of  $\theta$  and  $k_v$  on  $F_{dv} / F_{ds}$ .

**4. CONCLUSIONS**

An analytical model has been developed that is able to describe changes in friction force and drive force during sliding motion of a body over an in-plane vibrating surface. The new model has been validated against existing models for longitudinal and transverse vibration, and can also be used to simulate any mode of coupled vibration.

The model can be used in any three-dimensional domain, such as the flat-on-flat contact domain used in this paper where the normal contact pressure has been assumed constant, or a more complex case of gears for example, where the normal contact pressures change as gear teeth enter and exit the gear mesh to form multiple contacts simultaneously. In the case of flat-on-flat contact, the greatest reduction in  $F_d$  is achieved by longitudinal vibration, however, friction is in nearly all engineering problems and in other applications a different mode of vibration may be more beneficial.

**REFERENCES**

[1] Gutowski, P., & Leus, M. (2012). The effect of longitudinal tangential vibrations on friction and driving forces in sliding motion. *Tribology International*, 55(6), 108-118.

[2] Gutowski, P., & Leus, M. (2015). Computational model for friction force estimation in sliding motion at transverse tangential vibrations of elastic contact support. *Tribology International*, 90, 455-462.

[3] Chowdhury, M., Helali, M. (2008). The effect of amplitude of vibration on the coefficient of friction for different materials. *Tribology International*, 41(4), 307-314.

[4] Dahl, P. (1976). Solid friction damping of mechanical vibrations. *AIAA Journal*, 14(12), 1675-1682.