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Conditions for assuming a conformal contact to be non-conformal

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ABSTRACT – A transition condition from a conformal contact to a non-conformal contact is studied. A cylinder-hole and the Hertz contacts are considered as a conformal and a non-conformal contact, respectively. First, the theory of the contact between a cylinder and a hole is revisited. Its peak traction is compared with the Hertzian peak. In the case of contact with elastically similar materials, the non-conformal contact can be assumed until the angle of contact reaches almost 38% of the maximum possible angle of contact if the difference in the traction peak is set as 5%.

1. INTRODUCTION

According to the definition, a conformal contact means the size of the contact patch is comparable with that of the characteristic dimensions of the contacting bodies, whereas, a non-conformal contact is the term used when the contact size is assumed to be infinitesimal compared with the contacting body sizes [1]. The Hertz contact is a typical example of non-conformal contact, whereas a cylinder-to-hole contact shows a typical conformal contact configuration.

However, a generic question occurs; when is non-conformal contact changed to conformal contact? This is because a point contact initially takes place when a cylinder begins to make contact with a hole. As the force increases, the contact size (angle) is enlarged and eventually become a certain dimension, which constitutes a conformal contact. If the contact is non-conformal, it is relatively easy to analyse the contact and internal stresses because the fundamental assumption of the elasticity theory can be well applied (i.e., a far field stress is not affected by the stresses of the region of interest) [2]. Thus, it is useful to know the transition condition between the conformal and non-conformal contacts.

An actual example can be found in the nuclear fuels of a sodium cooled fast reactor (SFR). One end of the fuel rod is assembled to a support (mounting rail) by a cylinder-hole joint as illustrated in figure 1. During reactor operation, a sodium coolant flows upwards so a fuel rod tends to be lifted together with a buoyancy force against the dead weight of the rod. The rods vibrate owing to the flow. Reciprocal slipping occurs between the fuel rod end cap and mounting rail, which can cause wear at the contact surfaces. If the wear becomes severe, the rod may escape from the mounting rail which should be strictly prohibited for nuclear safety.

To analyse this kind of wear failure, the first step is to know the contact stresses. This work focuses on this problem and tries to develop a condition when a conformal contact can be assumed as a non-conformal contact. For this purpose, a cylinder-to-hole contact is used for a conformal configuration, whereas the Hertz contact is used for a non-conformal one. Figure 2 illustrates the geometry of the present conformal contact between a cylinder and a hole.

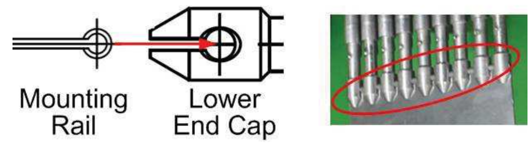

Figure 1 Conformal contact configuration of the SFR fuel rods.

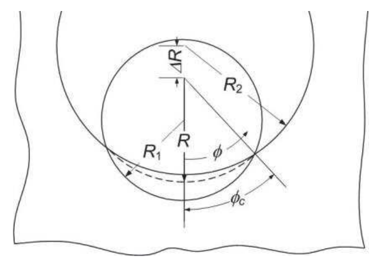

Figure 2 Geometrical model of a cylinder-hole contact.

2. CONFORMAL CONTACT THEORY

Contact tractions in the case of a cylinder-hole contact can be obtained from the following singular integro-differential equation [3].

$$\int_{-b}^{b}\frac{df(t)}{dt}\frac{dt}{t-y}+\frac{\lambda\pi}{1+\eta}\frac{f(y)}{1+y^2}=-\frac{4\xi}{\pi\left(1+y^2\right)^2}+\frac{B}{1+y^2} \qquad (1)$$

where $B=\frac{2\xi}{\pi}-\frac{2\eta}{1+\eta}\int_{-b}^{b}\frac{f(t)dt}{1+t^2}-\frac{\pi}{(1+\eta)}\frac{E_1^*\Delta R}{P}$, and $f(y)=R\,p(y)/P$, being the normalized contact normal traction of $p(y)$, which satisfies the following closure condition.

$$\int_{-b}^{b}\frac{1-t^2}{\left(1+t^2\right)^2}dt=\frac{1}{2}\,, \qquad (2)$$

whence, b and y are geometric parameters, and η, λ, ξ are material parameters defined as follows.

$$b=\tan(\phi_c/2),\ \ y=\tan(\phi/2),\ \ \eta=E_1^*/E_2^*\,, \qquad (3)$$

$$\lambda=\left(1-\nu_1^*\right)-\eta\left(1-\nu_2^*\right),\ \ \xi=\frac{\pi}{2}\frac{2(1+\eta)-\lambda}{1+\eta}\,. \qquad (4)$$

In equations (3) and (4), subscripts 1, 2 designate the materials of the cylinder and hole, respectively. The superscript, *, distinguishes the plane stress and plane strain conditions such that $E^* = E$, $\nu^* = \nu$ in the plane stress, and $E^* = E/(1-\nu^2)$, $\nu^* = \nu/(1-\nu)$ in the plane strain condition (subscripts 1, 2 are omitted here).

If we assume the materials of the cylinder and hole are the same, the normalized contact normal traction can be derived as follows.

$$f(y) = \frac{R\,p(y)}{P} = \frac{R\,p(\phi)}{P} = \frac{2}{\pi\sqrt{b^2+1}}\frac{\sqrt{b^2-y^2}}{1+y^2} + \frac{1}{2\pi b^2\left(b^2+1\right)}\log\left[\frac{\sqrt{b^2+1}+\sqrt{b^2-y^2}}{\sqrt{b^2+1}-\sqrt{b^2-y^2}}\right]. \tag{5}$$

The half contact angle, ϕ_c, is obtained from b (see equation (3)), which is evaluated from the following equation.

$$\frac{E^* \Delta R}{P} = \frac{2}{\pi}\frac{1-b^2}{b^2} - \frac{I}{\pi^2 b^2\left(b^2+1\right)}, \tag{6}$$

$$\text{where } I = \int_{-b}^{b} \log\left[\frac{\sqrt{b^2+1}+\sqrt{b^2-t^2}}{\sqrt{b^2+1}-\sqrt{b^2-t^2}}\right]\frac{dt}{t^2+1}. \tag{7}$$

On the other hand, the contact normal traction can be derived as follows if the contact traction is the Hertzian.

$$\frac{R\,p(\phi)}{P} = f(y) = \frac{2}{\pi\,\phi_c}\sqrt{1-\left(\frac{\phi}{\phi_c}\right)^2}. \tag{8}$$

3. CONDITION FROM THE CONFORMAL TO NON-CONFORMAL CONTACTS

$f(y)$ in equation (5) has a semi-elliptical shape just like the Hertzian profile of equation (8). Therefore, the peak values of $f(y)$ in equations (5) and (8) are to be compared to find a condition in which the conformal contact traction can be assumed as the non-conformal contact (i.e., the Hertzian) traction. The result is given in figure 3. In this work, it is accepted that the conformal contact can be regarded as the non-conformal one when the difference in their peak stresses is less than 5%. As a result, this condition takes place when $\phi_c \leq 35.57°$, which is also depicted with a vertical line in figure 3.

On the other hand, it is easy to know that ϕ_c depends on the elastic modulus (E), the difference in the radii of the cylinder and hole (ΔR) and contact force (P). Thus, it is meaningful from an engineering viewpoint to have a relationship between those parameters within the range of $0° \leq \phi_c \leq 35°$ where the non-conformal contact can be assumed.

This result is provided in figure 4. It is found that the contact angle decreases when E and/or ΔR increase and/or P decrease, which is logical. When $\phi_c = 1°$ and $35°$, $E^*\Delta R/P$ = 8358.22 and 5.49. $E^*\Delta R/P$ dramatically decreases until ϕ_c increases up to 10°. Beyond that, the influence of $E^*\Delta R/P$ on ϕ_c is considerable. As an example, if the material of the cylinder and hole is type 304 stainless steel (E = 193 GPa, ν = 0.29) and ΔR = 1 mm, the transition from the non-conformal to conformal condition takes place when P > 38.38 kN (plane strain case). Similar calculations can be done for engineering purposes.

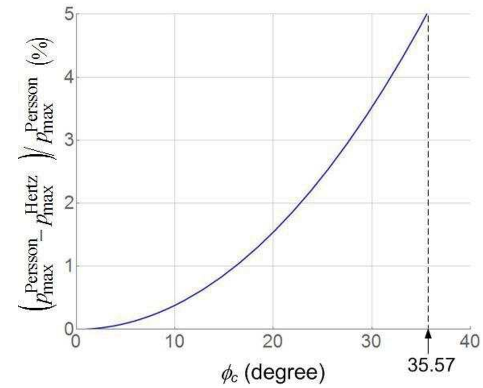

Figure 3 Ratio of the difference in the contact traction peak.

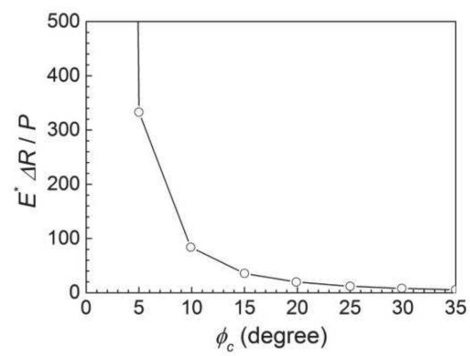

Figure 4 Relationship between $E^*\Delta R/P$ and ϕ_c.

4. SUMMARY

The condition of assuming a conformal contact to be non-conformal is investigated using a cylinder-hole contact geometry and the peak of the normal traction. The non-conformal contact can be assumed when the contact angle between the cylinder and hole of elastically similar materials is less than around 70°. This is unexpectedly large because it is almost 40% of 180°, which is the maximum angle of contact for this contact configuration. The influence of the elastic modulus, radii difference and contact force (i.e., $E^*\Delta R/P$) on the contact angle is considerable especially when the contact angle is within the range of 20°-70°.

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