Proceedings of Asia International Conference on Tribology 2018, pp. 283-284, September 2018

Application of equivalent stress method in finite-length elastic space with the consideration of two free end surfaces

Wang Zhijian, Chen Xiaoyang, Shen Xuejin*

School of Mechatronics Engineering and Automation, Shanghai University, Shanghai, China.

*Corresponding e-mail: shenxj@t.shu.edu.cn

Keywords: Free end surfaces; finite-length line contact; equivalent stress

ABSTRACT – In order to investigate the effect of the free end surfaces of finite-length elastic space on internal stresses and von-Mises stress, the equivalent stress method was presented which can effectively solve the free-edge problems and consider multi-direction stresses simultaneously. The results obtained by the present method are basically the same as the results of FEM analysis which represent the equivalent stress method is reliable.

1. INTRODUCTION

The contact problem of finite-length elastic space has been an interesting research topic since last century. In the early work of Hertz, a major assumption is made in modeling the contact bodies as equal length or infinite length. As Johnson [1] mentioned, although the contact stresses over the majority of the length of the cylinder are predicted accurately by Hertz theory, significant deviations occur close to the ends. The free boundaries of the ends have a significant influence. Wang et al.[2] applied the equivalent inclusion method to handle problems of two joined quarter spaces, which were treated as a class of special cases of two joined quarter spaces by setting Young's modulus of one of the materials to zero. Zhang et al [3,4] followed the Zhang's work [5] and extended the overlapping method and matrix formulation to finite-length elastic space problem. However, the shear stresses on two free end surfaces still exist.

Up to now, the solution of fully satisfying two free end surfaces for finite-length elastic space problems has not been provided. In this paper, Guo et al [6] recently presented an explicit matrix algorithmic for solving three-dimension wedge problems. The methodology is based on the concept of overlapping two half-spaces formed by the surfaces of a wedge and all calculations are based on half-space equivalent loads. In this paper, based on this idea, a new equivalent stress method will be presented to solve the finite-length elastic space problems. Using this method, the normal stress and shear stress can be considered simultaneously, and the planes of finite-length elastic space also can be loaded simultaneously.

2. THE GENERAL MODEL

In the finite-length elastic space, three distributed pressures P, P_1 and P_2 are loaded at z = 0 plane. In order to meet the boundary conditions of two free end surfaces, it can be equivalent to nine distributed pressures $P1_{xx}$, $P1_{xy}$, $P1_{xz}$, P_{zz}, P_{zx}, P_{zy}, $P2_{xx}$, $P2_{xy}$, $P2_{xz}$, which loaded at three different half spaces respectively, as shown in Figure 1. If these nine distributed pressures can satisfy the following nine equations simultaneously, the boundary conditions of two free end surfaces can be fully met. It should be noted that here the contact area is assumed to be far from the free surfaces #3 and #4. Thus, the effect of free surfaces in y direction can be neglected.

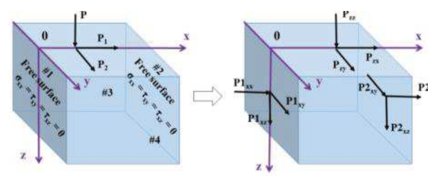


Figure 1 The solution of finite-length elastic space.

Surface #1:

$$P1_{xx} - P1'_{xx} - P1^*_{xx} = 0 \quad (1)$$

$$P1_{xy} - P1'_{xy} - P1^*_{xy} = 0 \quad (2)$$

$$P1_{xz} - P1'_{xz} - P1^*_{xz} = 0 \quad (3)$$

Where $P1'_{xx}$, $P1'_{xy}$, $P1'_{xz}$ are the internal stresses in surface #1 produced by the P_{zz}, P_{zx}, P_{zy}. $P1^*_{xx}, P1^*_{xy}, P1^*_{xz}$ are the internal stresses in surface #1 produced by the $P2_{xx}, P2_{xy}, P2_{xz}$.

Surface #2:

$$P2_{xx} + P2'_{xx} + P2^*_{xx} = 0 \quad (4)$$

$$P2_{xy} + P2'_{xy} + P2^*_{xy} = 0 \quad (5)$$

$$P2_{xz} + P2'_{xz} + P2^*_{xz} = 0 \quad (6)$$

where $P2'_{xx}, P2'_{xy}, P2'_{xz}$ are the internal stresses in surface #2 produced by the P_{zz}, P_{zx}, P_{zy}. $P2^*_{xx}, P2^*_{xy}, P2^*_{xz}$ are the internal stresses in surface #2 produced by the $P1_{xx}$, $P1_{xy}, P1_{xz}$.

Surface z = 0:

$$P_{zz} - P1'_{zz} - P2'_{zz} = P \quad (7)$$

$$P_{zx} - P1'_{zx} - P2'_{zx} = P_1 \quad (8)$$

$$P_{zy} - P1'_{zy} - P2'_{zy} = P_2 \quad (9)$$

where $P1'_{zz}, P1'_{zy}, P1'_{zx}$ are the internal stresses in surface z = 0 which produced by $P1_{xx}, P1_{xy}, P1_{xz}$, and $P2'_{zz}, P2'_{zy}$, $P2'_{zx}$ are the internal stresses in surface z = 0 which produced by $P2_{xx}, P2_{xy}, P2_{xz}$.

The equations (1~9) form the system of linear equations which has nine unknown distributed pressures. This system of linear equations can be solved by Gaussian elimination with backward substitution or iterative method. By Gaussian elimination method, the final analytical expressions can be obtained. However, sometimes these expressions of unknown variable are so complex that many computers cannot store matrices of such size, making computer manipulation impossible. Iterative method provides the possibility for solving this system. Here, Jacobi iterative method is applied.

3. RESULTS AND DISCUSSION

The results of finite-length elastic space from the proposed method are compared with FEM results. The commercial software package ABAQUS v6.13 was used to contact three-dimensional finite element analysis. An ideal line contact pressure distribution is applied to the top surface (z = 0) of the finite-length elastic space of Figure 2. The detailed expression is shown in equation 10, where p_0 is the maximum Hertzian pressure and a is the contact half width. The material properties of finite-length elastic space are elastic modulus 206000 MPa and Poisson ratio 0.3. The solution domain includes (10a, 5a, and 5a).

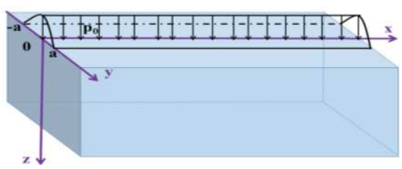


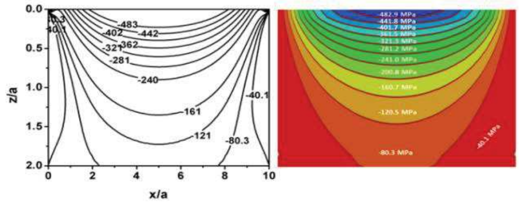
Figure 2 The model of finite-length elastic space.

$$p(x,y) = p_0 \frac{\sqrt{a^2 - y^2}}{a}, \quad |y| \le a$$
$$p(x,y) = 0, \quad |y| > a \tag{10}$$

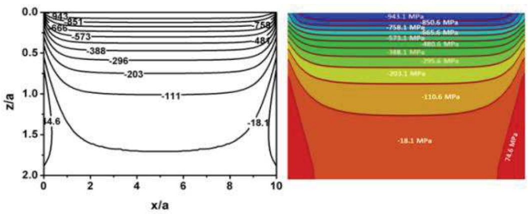
Figure 3 compares the results of the normal stresses distribution and von-Mises stresses by the equivalent stress method and FEM analysis method. It can be found that the magnitude and trend obtained by two methods are basically the same. The difference mainly exists near the edge of the contact area which may be caused by the density of the mesh.

4. SUMMARY

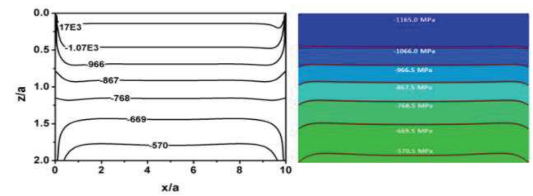
The equivalent stress method was introduced to solve the finite-length elastic space problems in this paper. The results obtained by the present method are compared with the results of FEM analysis. The results are basically the same which represents the equivalent stress method is reliable. In addition, this method can be further extended to solve the contact problem and any free-edge problem.



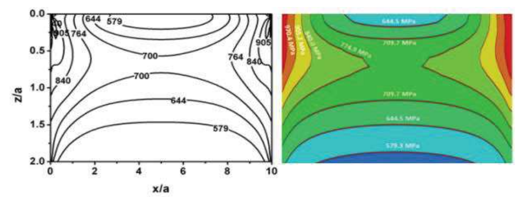
(a) Contours of σ_{xx} in the (x,z)-plane.



(b) Contours of σ_{yy} in the (x,z)-plane.



(c) Contours of σ_{zz} in the (x,z)-plane.



(d) Contours of von-Mises stress in the (x,z)-plane.

Figure 3 Comparison of results obtained by the present method (left) and FEM analysis (right).

REFERENCES

[1] Johnson, K. L., & Johnson, K. L. (1987). *Contact mechanics*. Cambridge university press.

[2] Wang, Z., Jin, X., Keer, L. M., & Wang, Q. (2012). Numerical methods for contact between two joined quarter spaces and a rigid sphere. *International Journal of Solids and Structures*, *49*(18), 2515-2527.

[3] Zhang, H., Wang, W., Zhang, S., & Zhao, Z. (2016). Modeling of finite-length line contact problem with consideration of two free-end surfaces. *Journal of Tribology*, *138*(2), 021402.

[4] Zhang, H., Wang, W., Zhang, S., & Zhao, Z. (2017). Modeling of elastic finite-length space rolling-sliding contact problem. *Tribology International*, *113*, 224-237.

[5] Zhang, Z. M., Wang, W., & Wong, P. L. (2013). An explicit solution for the elastic quarter-space problem in matrix formulation. *International Journal of Solids and Structures*, *50*(6), 976-980.

[6] Guo, L., Zhang, Z. M., Wang, W., & Wong, P. L. (2017). An explicit matrix algorithm for solving three-dimensional elastic wedge under surface loads. *International Journal of Solids and Structures*, *128*, 231-242.