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STABILITY OF A RIGID ROTOR IN JOURNAL BEARINGS

A. A. Zakaria and J. I. Inayat-Hussain

Department of Mechanical Engineering
Universiti Tenaga Nasional, Jalan IKRAM-UNITEN
43000 Kajang, Selangor Darul Ehsan, Malaysia
E-mail: aazrul@uniten.edu.my

ABSTRACT

Linear stability analysis of an unbalanced rigid rotor supported by nonlinear journal bearings is undertaken using the Floquet theory, and verified using the numerical integration method. For small magnitude of rotor unbalance and operation of journal bearing in the lightly to moderately loaded regime, the threshold of instability was found to be almost similar to that predicted for the balanced rotor, whilst for moderate to large values of rotor unbalance, the instability threshold values were observed to be much higher than that predicted for the balanced rotor. For journal bearings operation in the highly loaded regime, the instability threshold values of the unbalanced rotor were observed to be lower than that of the balanced rotor. The rotor unbalance magnitude required to suppress journal bearing instability is, however, considerably large, exceeding the permissible unbalance level for rigid rotor.

Keywords: Floquet theory, Journal bearings, Rotordynamics, Stability

1. INTRODUCTION

Journal bearings are strongly nonlinear machine element and they significantly influence the dynamic characteristics of the rotating machinery that they support. In the design and operation of rotating machinery, its stability is of utmost importance to ensure safe operation of these machines at their rated speed and load. Linear analysis is normally used to determine the stability of rotating machinery in journal bearings. In this analysis, the equations of motion of the rotor-bearing system are linearized about its static equilibrium position and the stability of this equilibrium position subjected to small disturbance is evaluated. This method has two drawbacks, namely the stability of the rotor is only valid for small motion about the static equilibrium position, and the unbalance force are totally neglected in this analysis. The effect of rotor unbalance cannot be neglected because it is not possible to have a perfectly balanced rotor in practical applications. A brief review of past work on the stability of balanced and unbalanced rotors is presented below.

Holmes (1970) examined the nonlinear performance of turbine bearings by numerically integrating the equations of motion of a rigid rotor mounted on cavitated short journal bearings. He recommended that a correction factor be applied to linear bearing coefficients because of the significance of non-linear effect as the eccentricity ratio of the bearing increased. He further suggested that for turbo-rotors, which typically have eccentricity ratios of 0.7 and 0.4, linearity is valid for peak-to-peak vibration values of about one-third of the radial clearance and full radial clearance, respectively.Holmes et al. (1978) investigated the vibration response of a rigid shaft supported by short journal bearings. They revealed two distinct behaviors of the journal motion, which depended on the eccentricity ratio of the journal. For most of the operating conditions investigated in this work, the motion of the journal was found to be asymptotically periodic with components principally at synchronous and half synchronous frequency. For operation of the rotor at high eccentricity ratio, aperiodic behavior was observed where the motion of the journal was complex and did not settle to a limit cycle. Barret et al. (1976) investigated the effect of unbalance and ambient pressure on a journal bearing for both cases of operation below and above the linear stability threshold speed. They found that, for operation above the stability threshold speed, increasing the rotor unbalance magnitude resulted in the decrease of rotor amplitude motion and force transmitted. This observation suggested the possibility of suppressing oil whirl in journal bearing by increasing the magnitude of the rotor unbalance force. Bannister and Makdissy (1980) examined the effect of unbalance on the stability and non-synchronous whirling of a rotor supported by journal bearings. They found that an increase in unbalance magnitude improved the stability of the rotor-bearing system, concurring with the findings of Barrett et al. (1976). The work of Lund and Nielsen (1980) on the stability of an unbalanced rigid rotor mounted on short journal bearings also revealed that rotor unbalance raised the threshold of instability especially at high values of the modified Sommerfeld number. Khonsari and Chang (1993) utilized a model of a perfectly balanced shaft supported by two identical cavitated journal bearings to investigate the stability boundary of these nonlinear bearings. Numerical results showed that the initial conditions for the eccentricity and attitude angle of the journal in the bearing were important parameters that influenced the stability of the system. They further showed the possible existence of unstable orbits for values of the rotor-bearing system operating parameters which had been determined to be stable based on the linearized stability theory. Wang and Khonsari (2006) utilized the Hopf bifurcation theory to predict the stability envelope of a rigid rotor mounted on journal bearings. This method required less computational time to predict the stability envelope as compared to an earlier work reported in Khonsari and Wang (1993) that employed a trial-and-error method. El-Shafei et al. (2007) performed experimental work to investigate the effect of unbalance magnitude, oil supply pressure and coupling misalignment on the threshold of instability of a flexible rotor supported by journal bearings. They found that angular misalignment of the coupling was the most effective method to delay the onset of instability in the rotor-bearing system.

In the present work, linear stability analysis of an unbalanced rigid rotor supported by nonlinear journal bearings is undertaken using the Floquet theory, and verified using the numerical integration method. The threshold of instability of the unbalanced rotor, which is determined for a range of practical values of static eccentricity ratio and rotor unbalance magnitude, is compared to that of the balanced rotor, and the influence of the magnitude of rotor unbalance on the instability threshold is examined.

2. THEORETICAL TREATMENT

2.1 Equations of Motion

The rotor, which has a mass of $2m$, is mounted in two identical journal bearings. The purely static unbalance of the rotor is represented by the eccentricity (u) of its center of mass (G) from its geometric center of rotation, which in the case of the rigid rotor is identical to the geometric center of the journal, O_J. The Cartesian coordinates (X, Y) are fixed in space and centred at O_B, the bearing center. The notation and the coordinate frames used in the analysis of the rotor-bearing system are shown in Figure 1. In the derivation of the equations of motion of a rigid rotor in journal bearings, the following assumptions are made: (i) the rotor is rigid and symmetric, (ii) unbalance of the rotor is purely static and located in its midspan plane, (iii) gyroscopic effects are neglected, (iv) rotor motion in the axial direction is neglected, (v) short bearing approximation based on Reynolds equation for incompressible flow is valid, and (vi) cavitation is modeled as π-film and therefore the contribution below ambient pressure to the oil-film forces is neglected.

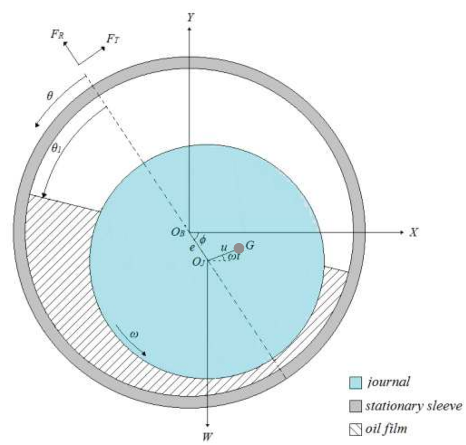


Figure 1 Journal bearing coordinate systems.

With the external forces acting on the journal that include the oil-film forces (F_X and F_Y), gravity, and unbalance force, the equations of motion of the journal center become

$$
\begin{aligned}
m\ddot{x} &= F_X + mu\omega^2 \cos \omega t \\
m\ddot{y} &= F_Y + mu\omega^2 \sin \omega t - mg
\end{aligned}
\tag{1}
$$

The pressure (P) distribution in a cavitated (π-film) short journal bearing derived from the Reynold's equation is given in rotating coordinates as

$$
P(\theta, z) = \left[\frac{6\mu}{h^3}\right]\left(z^2 - \frac{L^2}{4}\right) \cdot \left(\frac{\dot{\varepsilon}\cos\theta + \varepsilon(\dot{\phi} - 0.5\omega)\sin\theta}{(1+\varepsilon\cos\theta)^3}\right)
\tag{2}
$$

ε and $\dot{\varepsilon}$ respectively denote the radial displacement and velocity of the journal, and ω and $\dot{\phi}$ are respectively the journal's rotational and whirl velocity. θ is the angular coordinate measured from the position of maximum film thickness in the direction of rotor angular velocity. L denotes the length of the bearing, c the bearing's radial clearance, z the position in the axial direction of the bearing, and μ the dynamic viscosity of the lubricant. The resulting oil-film forces due to the motion of the journal in a bearing are usually expressed naturally in polar coordinates.

The forces in the polar coordinates can be easily transformed into the Cartesian coordinates by the following equations.

$$\begin{aligned} F_X &= F_R \cos\phi - F_T \sin\phi \\ F_Y &= F_R \sin\phi + F_T \cos\phi \end{aligned} \tag{3}$$

F_R and F_T are the radial and tangential oil-film forces, respectively. These forces are obtained by integrating the pressure distribution, given in (2), over the entire bearing surface, and can be expressed as functions of $(\varepsilon, \dot{\varepsilon}, \dot{\phi})$ by Equation (4).

$$\begin{aligned} F_R &= -\frac{\mu R L^3}{c^2}\left[I_1\dot{\varepsilon} + I_2\varepsilon(\dot{\phi} - 0.5\omega)\right] \\ F_T &= -\frac{\mu R L^3}{c^2}\left[I_2\dot{\varepsilon} + I_3\varepsilon(\dot{\phi} - 0.5\omega)\right] \end{aligned} \tag{4}$$

where,

$$\begin{aligned} I_1 &= \int_{\theta_1}^{\theta_1+\pi} \frac{\cos^2\theta}{(1+\varepsilon\cos\theta)^3} d\theta \\ I_2 &= \int_{\theta_1}^{\theta_1+\pi} \frac{\sin\theta\cos\theta}{(1+\varepsilon\cos\theta)^3} d\theta \\ I_3 &= \int_{\theta_1}^{\theta_1+\pi} \frac{\sin^2\theta}{(1+\varepsilon\cos\theta)^3} d\theta \\ \theta_1 &= \tan^{-1}\left(-\frac{\dot{\varepsilon}}{\varepsilon(\dot{\phi} - 0.5\omega)}\right) \end{aligned} \tag{5}$$

The integrals I_1, I_2 and I_3 can be evaluated analytically in closed form, (Booker, 1965). θ_1 denotes the angular position of the start of positive pressure region measured from the position of maximum oil film thickness in the direction of rotor angular speed. Inserting Equations (4) and (5) into Equation (3), and the result into Equation (1), and dividing the resulting equation by $m\omega^2 c$, and substituting the appropriate non-dimensional parameters into the final equation, we obtain the non-dimensional governing equations for a rigid rotor mounted in cavitated short journal bearings.

$$\begin{aligned} x'' &= U\cos\tau - \frac{1}{M}\begin{bmatrix} \frac{x}{\varepsilon}\left[I_1\varepsilon' + I_2\varepsilon(\phi' - 0.5)\right] - \\ \frac{y}{\varepsilon}\left[I_2\varepsilon' + I_3\varepsilon(\phi' - 0.5)\right] \end{bmatrix} \\ y'' &= U\sin\tau - \frac{1}{M}\begin{bmatrix} \frac{y}{\varepsilon}\left[I_1\varepsilon' + I_2\varepsilon(\phi' - 0.5)\right] + \\ \frac{x}{\varepsilon}\left[I_2\varepsilon' + I_3\varepsilon(\phi' - 0.5)\right] \end{bmatrix} - \frac{1}{\sigma M} \end{aligned} \tag{6}$$

where,

$$\sigma = \frac{\mu\omega R L^3}{4Wc^3}$$

$$M = \frac{c\omega^2}{\sigma g}$$

$$U = \frac{u}{c}$$

The modified Sommerfeld number (σ), a parameter that is widely used as a characteristic number for journal bearing performance, determines the static equilibrium position of the journal in the bearing. It depends on the lubricant viscosity (μ), journal rotational velocity (ω), journal radius (R) and length (L), bearing radial clearance (c) and the bearing load (W). Higher loads and lower speeds will result in a lower σ, whilst lighter loads and higher speeds results in higher σ. The parameter M is the dimensionless mass of the journal. The product σM is known as the journal mass parameter. The journal mass parameter, which is also known as the stability parameter, is proportional to the square of the rotor's operating speed. The unbalance parameter (U), which is a measure of the rotor unbalance, is defined as the ratio of the eccentricity (u) of the rotor center of gravity (G) from its geometric center of rotation, to the radial clearance of the bearing (c). τ is the non-dimensional time.

2.2 Floquet Analysis

The stability of periodic solutions can be examined using Floquet theory. The state variables of the periodic solution are perturbed about its steady state, resulting in a system of linearized equations with periodically varying coefficient. The stability of the original system of nonlinear equations is then determined by the eigenvalues of the monodromy matrix, which is obtained from the solutions of these linearized equations over one period, with the identity matrix as the initial conditions. The eigenvalues of the monodromy matrix are known as Floquet multipliers. The periodic solution is stable if the magnitude of all the multipliers is less than one. The periodic solution loses its stability when at least one of the multipliers has a magnitude greater than one. Detailed expositions of the Floquet theory are found in Nayfeh and Balachandran (1993) and Seydel (2009).

The Floquet theory was used to evaluate the threshold of instability of the periodic solutions of the rotor-bearing system for a range of rotor unbalance magnitudes and eccentricity ratios. The threshold of instability is determined from the corresponding value of M when the magnitude of the leading Floquet multiplier traverses the line of unity. An example, shown in Figure 2, which depicts the magnitude of the leading Floquet multiplier for $\varepsilon = 0.3$ and $U = 0.2$, gives the threshold of instability of $M = 6.5$ (Note that the exact value of the threshold of instability lies between $M = 6.5$ and $M = 7$. Since the simulation was undertaken at interval of 0.5, the value of M immediately before the rotor becomes unstable is considered as the threshold of instability).

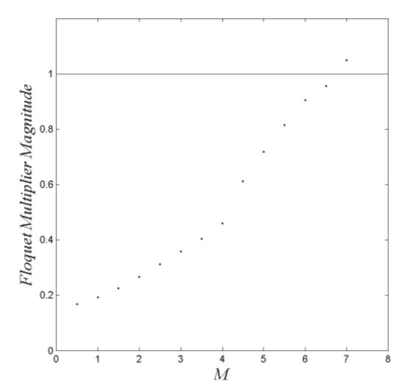


Figure 2 Magnitude of leading Floquet multiplier for $\varepsilon = 0.3$, $U = 0.2$ and the critical value of M is 6.5.

3. RESULTS AND DISCUSSION

For a given value of the unbalance parameter U, the instability threshold of the dimensionless journal mass M is determined for increasing values of the eccentricity ratio ε. ε, which is the ratio of the journal's static equilibrium position to the bearing's radial clearance, can be related to the modified Sommerfeld number σ (Lund and Saibel, 1967). The stability parameter σM is then plotted against σ to obtain the stability curve. The lower limit of ε is set to 0.1, determined by the validity of the Reynolds boundary condition, (Frene et al., 1997). The upper limit of ε, on the other hand, is set to 0.75, as practical operation of such bearings at values exceeding this is not recommended to avoid possibility of rubbing between the journal and the stationary bearing sleeve.

The stability of the rotor for several unbalance magnitudes ranging from 0.05 to 0.4 was investigated in this work. The influence of unbalance magnitudes on the stability of the rotor is shown in Figure 3 for $U = 0.05$, 0.1 and 0.15 corresponding to small unbalance magnitude and Figure 4 for $U = 0.2$, 0.25, 0.3, 0.35 and 0.4 corresponding to moderate to large magnitudes of unbalance. As shown in Figure 3, the threshold of instability computed for the case of small unbalance magnitude is lower than that determined from the linear stability curve (corresponding to $U = 0.0$). The deviation is seen to be more significant at lower values of σ, which corresponds to journal bearing operation at higher values of eccentricity ($0.55 \leq \varepsilon \leq 0.75$).

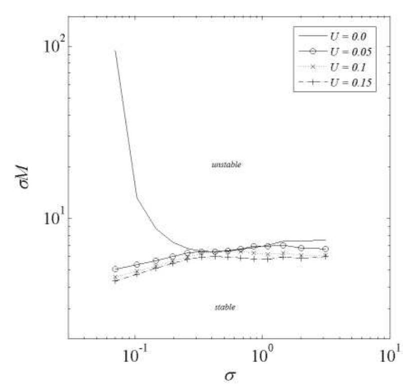


Figure 3 Stability curve for small unbalance magnitudes.

The stability curves for the case of moderate to high value of unbalance magnitudes, $0.2 \leq U \leq 0.4$, are shown in Figure 4. These stability curves showed a general trend of increasing instability threshold with the increase of unbalance magnitudes for moderate to large values of σ, which corresponds to lightly to moderately loaded operating regime of the journal bearing ($0.1 \leq \varepsilon \leq 0.5$). With the exception of $U = 0.2$ and 0.25, the curves representing other values of U indicated that the instability threshold was higher than that predicted for the case of the balanced rotor (U = 0.0) for large values of σ. For moderate values of σ, on the other hand, the threshold of instability for U = 0.35 and 0.4 was seen to be higher as compared to the threshold predicted for the balanced rotor.

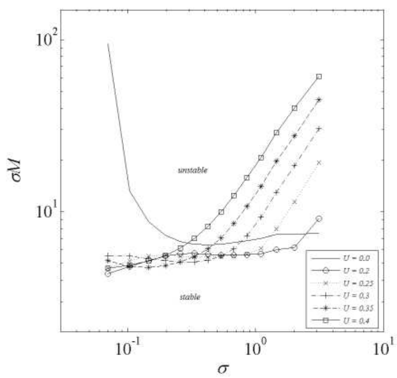
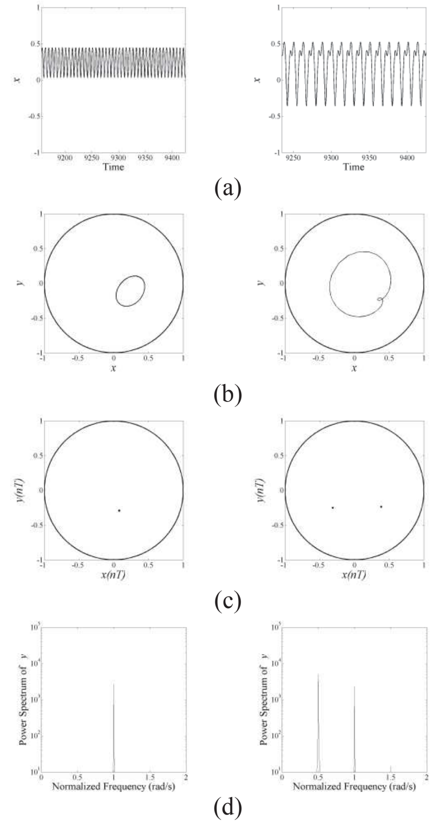


Figure 4 Stability curve for moderate to high unbalance magnitudes.

For the case of ε = 0.3 and U = 0.2, whose leading Floquet multiplier and stability map are respectively shown in Figures 2 and 4, the rotor response determined from direct numerical integration of the governing equations are presented using Poincaré map, time series, whirl orbit and power spectrum plots in Figure 4 for M = 6.5, corresponding to synchronous response and M = 7, corresponding to period-2 response. For this set of rotor-bearing system parameters, the loss of stability is due to period-doubling bifurcation as the value of M is increased from 6.5 to 7.

The results presented in Figures 3 and 4, which generally indicate that increasing rotor unbalance magnitude stabilizes an otherwise unstable rotor in journal bearings, concur with the findings of other authors; Barrett et al., (1976), Bannister and Makdissy (1980) and Lund and Nielsen (1980). A comparison with the stability curve of the balanced rotor showed that rotor unbalance increases the threshold of instability only for moderately to large values of σ, which corresponds to lightly to moderately loaded journal bearing operating regime. This fact is however not true for small values of σ, which corresponds to heavily loaded journal bearing operating regime, as clearly seen in Figures 3 and 4. The threshold of instability in this operating regime is much lower than that predicted for the balanced rotor.



M = 6.5 M = 7

Figure 5 Rotor response for ε = 0.3, U = 0.2, M = 6.5 (stable) and M = 7 (unstable), (a) time series, (b) whirl orbit, (c) Poincaré map, (d) power spectrum.

4. CONCLUSIONS

The threshold of instability of the unbalanced rotor supported by nonlinear journal bearings was determined using Floquet analysis, and further verified using the numerical integration method. For small magnitude of rotor unbalance and operation in the lightly to moderately loaded regime, the threshold of instability was found to be almost similar to that predicted for the balanced rotor. For moderate to large values of rotor unbalance and operation in the same regime, however, the instability threshold values were observed to be much higher than that predicted for the balanced rotor. For journal bearings operation in the highly loaded regime, the instability threshold values of the unbalanced rotor were observed to be lower than that of the balanced rotor. Although it has been recognized that journal bearing instability can be suppressed by rotor unbalance force, this work has shown that the rotor unbalance magnitude required for this purpose is considerably large, exceeding the permissible unbalance level for rigid rotor.

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NOMENCLATURE

c	bearing radial clearance, m
F_R	oil-film force in radial direction, N
F_T	oil-film force in tangential direction, N
F_X	oil-film force in *X*-direction, N
F_Y	oil-film force in *Y*-direction, N
G	center gravity of rotor
g	gravitational acceleration, ms^{-2}
h	dimensionless oil-film thickness
L	bearing length, m
M	dimensionless mass of the journal
m	half mass of rotor, kg
P	pressure, Nm^{-2}
R	journal radius, m
U	unbalance parameter
W	bearing load, N
x	non-dimensional displacement of the center of the rotor in the *X*-direction
y	non-dimensional displacement of the center of the rotor in the *Y*-direction
z	position in the axial direction of the bearing
ε	eccentricity ratio
$\dot{\varepsilon}$	radial velocity of journal, $rads^{-1}$
θ	angular coordinate measured from the position of maximum film thickness, rad
θ_1	angular position of the start of pressure region measured from θ, rad
μ	dynamic viscosity of lubricant, Nsm^{-2}
σ	modified Sommerfeld number
τ	non-dimensional time
ϕ	angular position of line connecting eccentric position of the journal centre to the bearing centre, rad
$\dot{\phi}$	whirl velocity, $rads^{-1}$